

Production Clustering and Offshoring

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APPENDIX

A1. Proofs of Propositions

Proposition 2

PROOF:

Let's assume there is an optimal path A for τ_0 , an optimal path B for τ_1 , $\tau_0 > \tau_1$, and $MC(A, \tau_0, C) < MC(B, \tau_1, C)$, where C is a vector of the costs of production. Note that $MC(A, \tau_0, C) \geq MC(A, \tau_1, C)$ and by optimality of B : $MC(A, \tau_1, C) \geq MC(B, \tau_1, C)$. It follows that $MC(A, \tau_0, C) \geq MC(B, \tau_1, C)$, which contradicts the initial assumption.

Lemma 1

PROOF:

Let path A with transportation quantity $TQ(A)$ be chosen for $\tau = \tau_0$ and path B with transportation quantity $TQ(B)$ be chosen for $\tau = \tau_1$ and $\tau_0 > \tau_1$. Now assume that the transportation quantity is an increasing function of τ and hence $TQ(A) > TQ(B)$. Then given the choice that the firm made under τ_1 : $NTMC(B) + TQ(B)\tau_1 < NTMC(A) + TQ(A)\tau_1$ and under τ_0 : $NTMC(B) + TQ(B)\tau_0 > NTMC(A) + TQ(A)\tau_0$. Adding $TQ(B)(\tau_0 - \tau_1)$ to the first inequality, I get: $NTMC(B) + TQ(B)\tau_0 < NTMC(A) + TQ(A)\tau_1 + TQ(B)(\tau_0 - \tau_1) < NTMC(A) + TQ(A)\tau_1 + TQ(A)(\tau_0 - \tau_1) = NTMC(A) + TQ(A)\tau_0$ or $NTMC(B) + TQ(B)\tau_1 < NTMC(A) + TQ(A)\tau_1$, which contradicts the condition on optimality of A under τ_0 .

Proposition 3

PROOF:

Let's assume $\tau_0 > \tau_1$. Let A be an optimal path for $\tau = \tau_0$ and transportation quantity $TQ(A) > 0$. Then by Proposition ?? $MC(A, \tau_0) > MC(A, \tau_1)$. Let B an optimal path for τ_1 , then by definition of optimal path $MC(A, \tau_1) \geq MC(B, \tau_1)$, and hence $MC(A, \tau_0) > MC(B, \tau_1)$.

Proposition 4

PROOF:

Let A be an optimal path for τ_0 , B an optimal path for τ_1 , $\tau_0 > \tau_1$, and $A \neq B$. By definition of optimality and because of the uniqueness of optimal paths, $MC(A, \tau_0) < MC(B, \tau_0)$ and $MC(A, \tau_1) > MC(B, \tau_1)$. From Lemma ?? $\tau_1 TQ(A) < \tau_1 TQ(B)$. Assume $NTMC(A) < NTMC(B)$, then

$MC(A, \tau_1) = NTMC(A) + \tau_1 TQ(A) < NTMC(B) + \tau_1 TQ(B) = MC(B, \tau_1)$, which contradicts the optimality of B under τ_1 .

A2. Incomplete Trees

To write down the problem for an arbitrary tree, I need to enumerate production nodes. Every node has a unique index $\{i, b\}$ that represents at what stage i the part is produced and to what branch b it belongs. Production costs for a part from branch b , produced on stage i in country k , are then $a_{i,b,k}$.

Stage $i = 1$ corresponds to the most downstream stage of production and $i = N$ denotes the most upstream stage. In case two or more of the intermediate goods are assembled together, each of the corresponding nodes gets the same stage number i ; in addition, each of these nodes gets branch index b , which was not previously assigned to another branch.

I define n_b as the last stage of branch b ; I call n_b the length of branch b . In addition, for each stage i I introduce an assembly set $\Omega_{i,b}$, which is the set of branch indexes b of all parts produced on stage $i + 1$, connected to stage $\{i, b\}$. $\nu_{i,b}$ is a branch of a part produced at stage $i - 1$, a node that $\{i, b\}$ is connected to. B_i is a set of all branches present at stage i . I present an example of such enumeration in Figure A1.

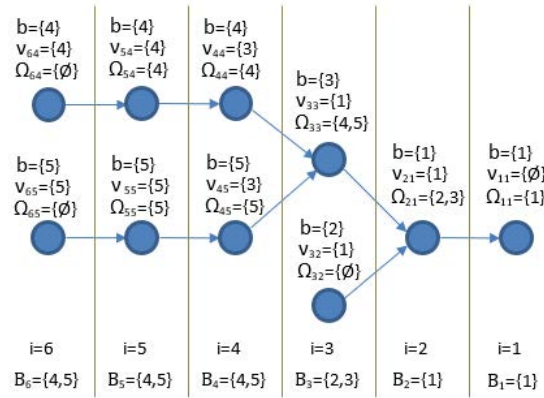


FIGURE A1. INCOMPLETE TREE NOTATION

$$MC = \min_{\{c_{i,b}\}} \sum_{i=1}^{\max\{n_b\}} \sum_{b \in B_i} \left(\sum_{k=1}^K \mathbf{1}(c_{i,b} = k) a_{i,b,k} + \tau T(c_{i,b}, c_{i-1\nu_{i,b}}) \right).$$

This expression differs from (??) due to more complex indexing structure of an

incomplete tree. The corresponding Bellman equation is

$$V_{i,b}(c_{i,b}) = \min_{c_{i,b} \in K} \left\{ \sum_{k=1}^K \mathbf{1}(c_{i,b} = k) a_{i,b,k} + \sum_{l \in \Omega_{i,b}} [\tau T(c_{i,b}, c_{i+1,l}) + V_{i+1,l}(c_{i+1,l})] \right\}.$$

A3. Clustering with Iceberg Trade Costs

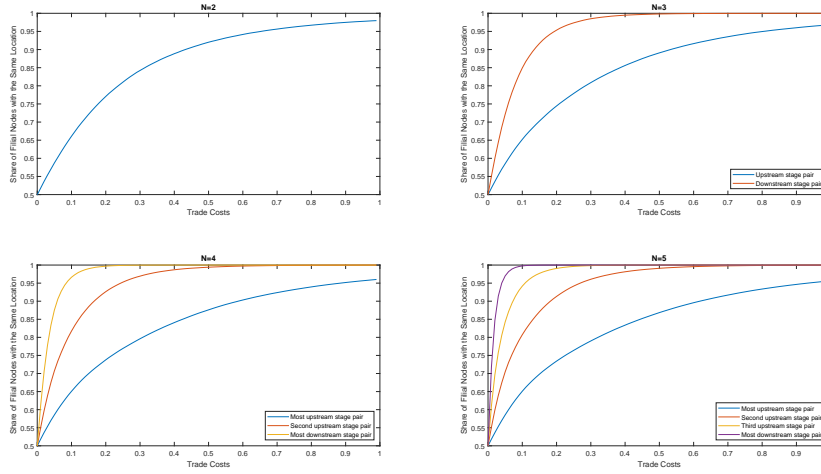


FIGURE A2. CLUSTERING AND TREE LENGTH: ICEBERG TRADE COSTS

A4. Elasticities

A5. Endogenous Wages

The problem presented above is the model of absolute advantage as there is no labor market. With a given supply of labor in each country L_j and endogenous wages that are determined through labor market clearing conditions, all countries will produce some parts no matter what production costs are.¹ I normalize the wage in country 1 to $w_1 = 1$. I assume that labor supply is perfectly inelastic and the firm has constant returns to scale production technology. The problem of every firm then looks like

¹As long as trade costs are not too high for a given firm.

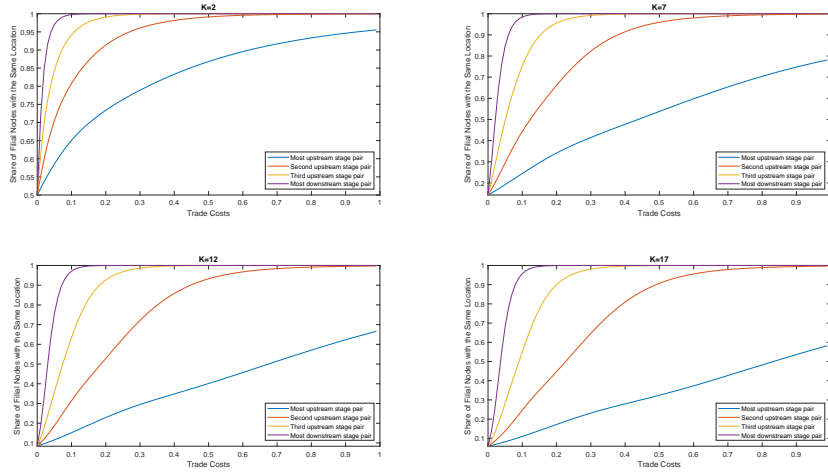


FIGURE A3. CLUSTERING AND THE NUMBER OF COUNTRIES: ICEBERG TRADE COSTS

$$(A1) \quad MC = \min_{c_{i,b}} \sum_{i=1}^N \sum_{b=1}^{M^{i-1}} \left(w_j \mathbf{1}(c_{i,b} = k) a_{i,b,k} + \tau T \left(c_{i,b}, c_{i-1, \lceil \frac{b}{M} \rceil} \right) \right),$$

and a firm's labor demand per unit produced is

$$L_{Dk} \equiv \sum_{i=1}^N \sum_{b=1}^{M^{i-1}} \mathbf{1}(c_{i,b} = k) a_{i,b,k} \text{ for } \forall k \in \{1, \dots, K\}.$$

Here for simplicity I assume that transportation services are performed by independent transport companies and do not affect domestic and foreign labor markets.

LEMMA A1: *Demand of the firm from country i L_{Di} for labor in country k is a nonincreasing function of w_k .*

PROOF:

Let the wage in country k decrease, while all other wages remain constant: $w_k^A > w_k^B$ and $w_{j \neq k}^A = w_{j \neq k}^B = w_{j \neq k}$. Let A and B be optimal paths under wage schedules w^A and w^B . In case $A = B$, $L_{Dk}^A = L_{Dk}^B$. Now consider the case $A \neq B$. Then because of the optimality of A and B : (a) $MC(A, w^A) < MC(A, w^B)$

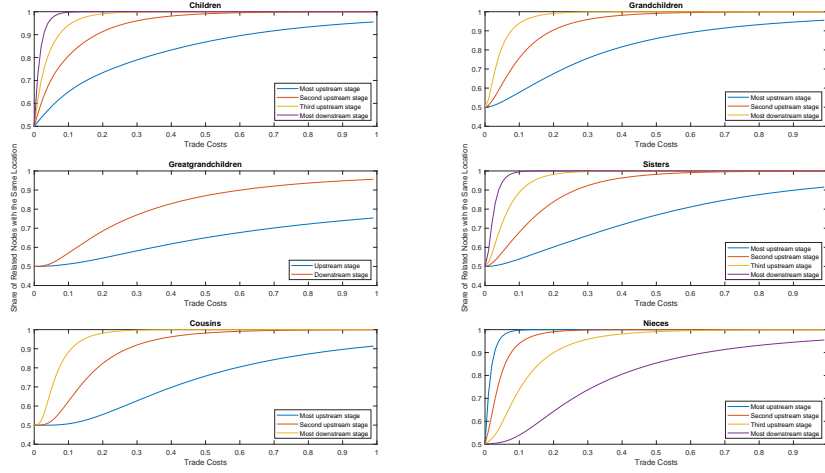


FIGURE A4. DIRECT AND INDIRECT CLUSTERING: ICEBERG TRADE COSTS

and (b) $MC(B, w^B) < MC(A, w^B)$. Let $\Delta^{\tau VT} \equiv \tau VT(A) - \tau VT(B)$, and $\Delta^L \equiv \sum_{j \neq k} w_j (L_{Dj}^A - L_{Dj}^B)$. Then (a) and (b) can be rewritten as: $L_k^A w_k^A - L_k^B w_k^A + \Delta^L + \Delta^{\tau VT} < 0$, and $L_k^A w_k^B - L_k^B w_k^B + \Delta^L + \Delta^{\tau VT} > 0$, subtracting the first inequality from the second obtains: $(L_{Dk}^A - L_{Dk}^B) (w_k^B - w_k^A) > 0$, and then $L_{Dk}^A < L_{Dk}^B$.

Note that if the firm changes its optimal path, then L_{Dk} is decreasing in w_k .

PROPOSITION A1: *There exists a wage schedule that clears the labor market. In a two-country case, this schedule is unique.*

PROOF:

Existence:

The world economy can be considered as an exchange economy with M agents, where labor supply in country k is the endowment of good k and wage in country k is the price of this good. Then from Lemma ?? demand of each agent for each good is nondecreasing in price of this good, so by proposition 17.C.1 in Mas-Colell et al. (1995) an equilibrium exists.

Uniqueness for the case of two countries:

A firm's relative demand for labor $\frac{L}{L^*}$ is a nonincreasing function of the relative wage w . Every firm takes the wage as given, but decisions of the firm determine the wage through market clearing condition. Here once again I apply the revealed preferences argument. Let's assume there is path A with $\sum_{i=1}^N c_i a_{wi} = R_{WA}$ and $\sum_{i=1}^N (1-c)_i a_{Ei} = R_{EA}$ that was chosen for $w = w_0$ and there is path

B with $\sum_{i=1}^N c_i a_{Ni} = R_{WB}$ and $\sum_{i=1}^N (1 - c)_i a_{Si} = R_{EB}$ that was chosen for $w = w_1$; $w_1 > w_0$ and $R_{WB} > R_{WA}$. Let function $NPC(Y)$ be a value of nonproduction costs for path Y , then given the choice that the firm made under w_0 : $NPC(A) + w_0 R_{WA} + R_{NA} < NPC(B) + w_0 R_{WB} + R_{EB}$ and under w_1 : $NPC(A) + w_1 R_{WA} + R_{EA} > NPC(B) + w_1 R_{WB} + R_{EB}$. Adding $R_{WA}(w_1 - w_0)$ to the both parts of the first inequality, I get: $NPC(A) + w_1 R_{WA} + R_{EA} < NPC(B) + w_0 R_{WB} + R_{EB} + R_{WA}(w_1 - w_0) < NPC(B) + w_1 R_{WB} + R_{EB} < NPC(B) + w_1 R_{WB} + R_{EB}$ or $NPC(A) + w_1 R_{WA} + R_{EA} < NPC(B) + w_1 R_{WB} + R_{EB}$, which contradicts the condition of optimality of B under w_1 .

For the case of multiple countries, proof of uniqueness of the equilibrium is nontrivial: decrease in the wage in one country can increase demand for labor in another country through the bridge FDI channel, similar to Proposition ???. As a result, the gross substitute property does not hold, and the uniqueness cannot be proven using the approach of Allen, Arkolakis and Li (2015).

*

REFERENCES

- Allen, Treb, Costas Arkolakis, and Xiangliang Li.** 2015. "On the existence and uniqueness of trade equilibria." *Manuscript, Yale Univ.*
- Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green, et al.** 1995. *Microeconomic theory*. Vol. 1, Oxford university press New York.

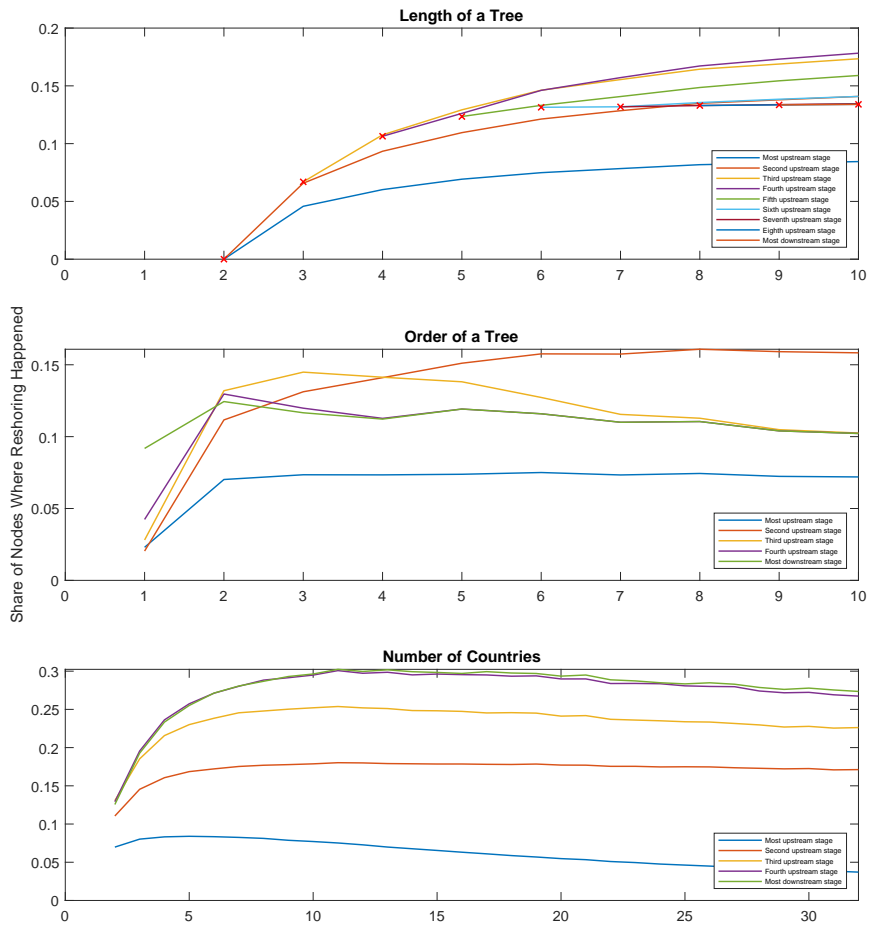


FIGURE A5. RESHORING: ICEBERG TRADE COSTS

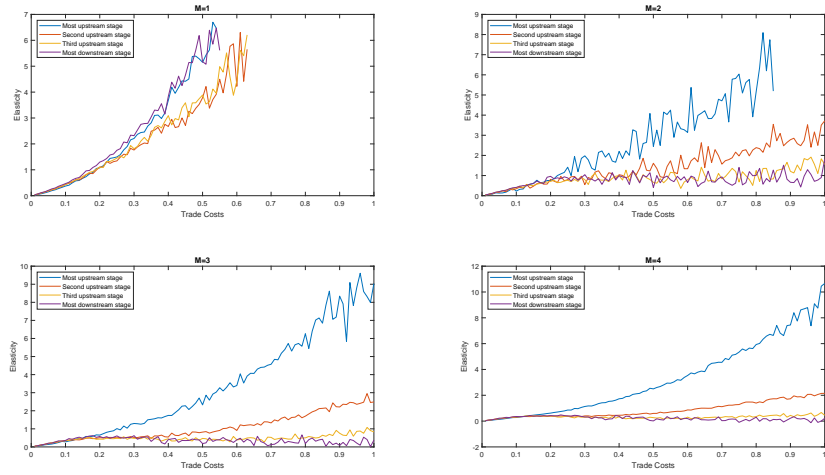


FIGURE A6. TRADE ELASTICITIES AND TREE ORDER

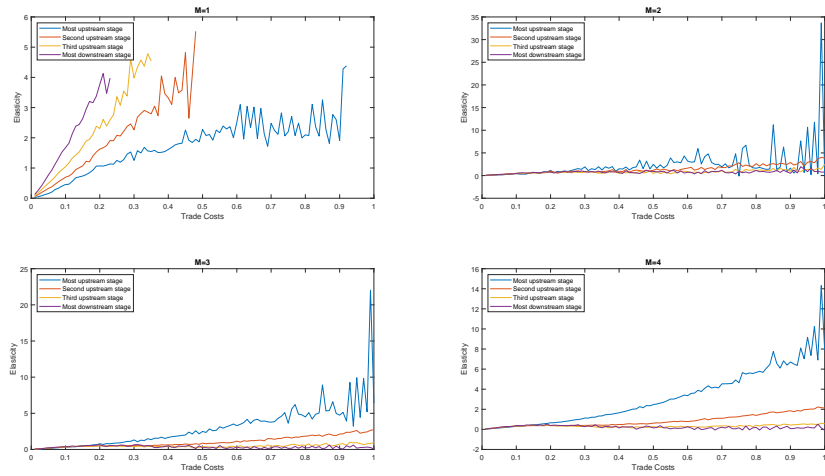


FIGURE A7. TRADE ELASTICITIES AND TREE ORDER: ICEBERG TRADE COSTS

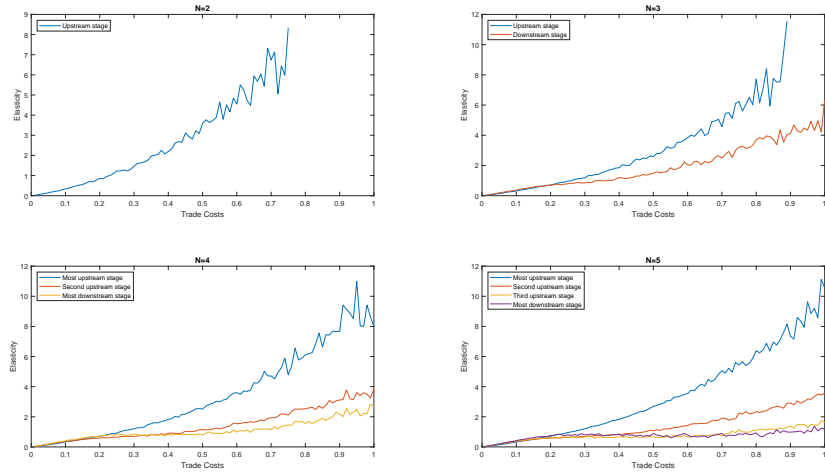


FIGURE A8. TRADE ELASTICITIES AND TREE LENGTH

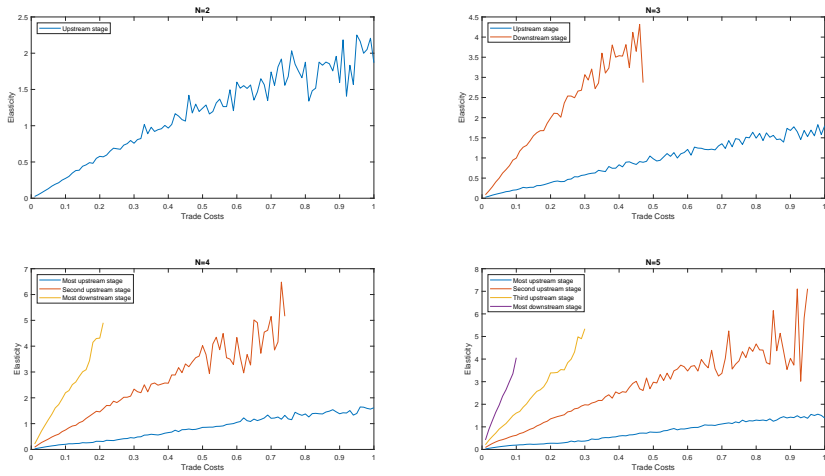


FIGURE A9. TRADE ELASTICITIES AND TREE LENGTH: ICEBERG TRADE COSTS

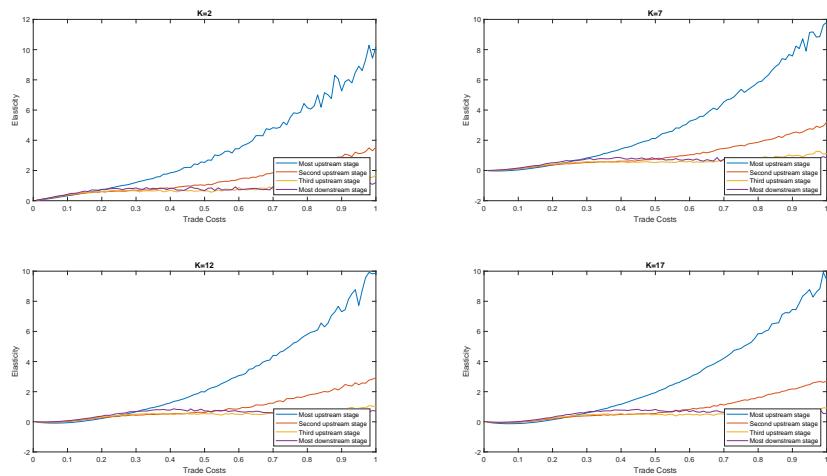


FIGURE A10. TRADE ELASTICITIES AND THE NUMBER OF COUNTRIES

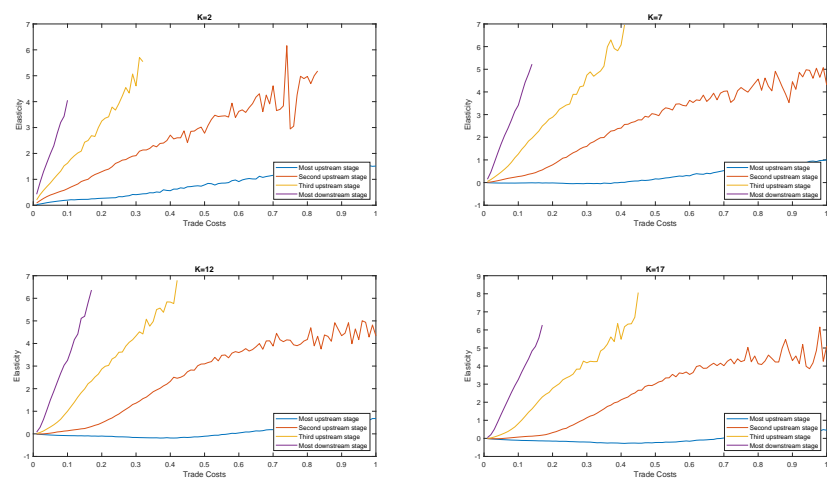


FIGURE A11. TRADE ELASTICITIES AND THE NUMBER OF COUNTRIES: ICEBERG TRADE COSTS